

# PRODUCTIVITY CHANGE AND THE DYNAMICS OF COST COMPETITIVENESS: A NONPARAMETRIC ANALYSIS

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**In this age of globalization nations across the world and states within a nation are trying to improve cost competitiveness in order to survive.**

**There is a continuing struggle to lower costs faster than the competitor**

**It can be described as a “race to the bottom”**

**‘It’s a slow sort of a country’, said the Rabbit.  
‘Here you must as fast as you can to stay where you  
are!’.**

***From Alice in the Wonderland***

## An Example of Changing Cost Competitiveness in US Manufacturing

Cost per \$ of Gross Output (1997 constant dollars)

	Average Cost								
Year	CA	IN	MA	MI	NJ	NY	NC	TX	VA
1992	0.701	0.737	0.684	0.787	0.697	0.651	0.669	0.765	0.645
1997	0.668	0.721	0.650	0.750	0.645	0.669	0.645	0.719	0.648
2002	0.697	0.726	0.670	0.786	0.675	0.631	0.630	0.789	0.627
2007	0.703	0.735	0.706	0.801	0.726	0.671	0.603	0.779	0.674

**North Carolina has become increasingly more cost competitive relative to the other states over the years**

	Average Cost Index								
Year	CA	IN	MA	MI	NJ	NY	NC	TX	VA
1997	0.954	0.979	0.951	0.953	0.925	1.027	0.963	0.940	1.004
2002	1.043	1.007	1.031	1.049	1.047	0.943	0.976	1.097	0.968
2007	1.009	1.013	1.053	1.018	1.075	1.064	0.958	0.988	1.075

**Change from previous Census Year**

**Cost competitiveness depends on**

**Productivity and**

**Input prices**

## Some previous studies:

### **Directly Related:**

**Lovell and Grifell-Tatje (2000)**

**Ray and Mukherjee (2000)**

### **Indirectly Related**

**Ray and Mukherjee (1996)**

Definitions:

Output:  $y$ ; Input:  $x$ ; Input price:  $w$

Actual cost:  $C = w'x$

Minimum cost:  $C^* = C(w;y)$

Average cost:

$$AC = \frac{C}{y}.$$

Comparing 2 firms: A and B

Cost Competitiveness Index:

$$CCI = \frac{AC_B}{AC_A} = \frac{\frac{C_B}{y_B}}{\frac{C_A}{y_A}}$$

$CCI > 1$  implies A is more cost competitive than B.

**Does a lower average cost always imply higher productivity?**

## Productivity and Average Cost

$$AC_A = \frac{w^{A'} x^A}{y_A} \quad AC_B = \frac{w^{B'} x^B}{y_B}$$

**Average cost Ratio:**

$$\begin{aligned} \frac{AC_A}{AC_B} &= \frac{\frac{w^{A, x^A}}{y_A}}{\frac{w^{B, x^B}}{y_B}} = \frac{\frac{y_B}{y^A}}{\frac{w^{B, x^B}}{w^{A, x^A}}} & \text{Alternatively,} & \frac{AC_A}{AC_B} = \frac{\frac{w^{A, x^A}}{y_A}}{\frac{w^{B, x^B}}{y_B}} = \frac{\frac{y_B}{y^A}}{\frac{w^{B, x^B}}{w^{A, x^A}}} \\ &= \frac{\frac{y_B}{y^A}}{\frac{w^{A, x^B}}{w^{A, x^A}}} \cdot \frac{w^{A, x^B}}{w^{B, x^B}} & & = \frac{\frac{y_B}{y^A}}{\frac{w^{B, x^B}}{w^{B, x^A}}} \cdot \frac{w^{A, x^A}}{w^{B, x^A}} \end{aligned}$$

Taking the geometric mean of the two

$$CCI = \frac{AC_A}{AC_B} = \frac{\frac{y_B}{y^A}}{\left[ \frac{w^A x^B}{w^A x^A} \cdot \frac{w^B x^B}{w^B x^A} \right]^{\frac{1}{2}}} \cdot \left[ \frac{w^A x^A}{w^B x^A} \cdot \frac{w^A x^B}{w^B x^B} \right]^{\frac{1}{2}}$$

Define

$$Q_x = \left[ \frac{w^A x^B}{w^A x^A} \cdot \frac{w^B x^B}{w^B x^A} \right]^{\frac{1}{2}}$$

$$W_x = \left[ \frac{w^B x^A}{w^A x^A} \cdot \frac{w^B x^B}{w^A x^B} \right]^{\frac{1}{2}} \quad Q_y = \frac{y_B}{y_A}$$

$$CCI = \frac{AC_B}{AC_A} = \left[ \frac{Q_y}{Q_x} \right] \cdot \frac{1}{W_x}$$

**The first factor on the right hand side is the inverse of Fisher Productivity Index.**

**CCI corresponds to the TFP of the firm only when the input price index equals unity**

## Change in Cost Competitiveness over Time:

**Period 0:**

**Firm A: output**  $y_A^0$

**Input price =**  $w_A^0$

**Cost =**  $C_A^0 = w_A^{0'} x_A^0$

**Minimum cost:**  $C_A^{0*} = C^0(w_A^0, y_A^0)$

**Firm B: output**  $y_B^0$

**Input price =**  $w_B^0$

**Cost =**  $C_B^0 = w_B^{0'} x_B^0$

**Minimum cost:**  $C_B^{0*} = C^0(w_B^0, y_B^0)$

## Change in Cost Competitiveness over Time:

### Period 1:

#### Firm A:

**Output** =  $y_A^1$

**Input price** =  $w_A^1$

**Cost** =  $C_A^1 = w_A^{1'} x_A^1$

**Minimum cost**  $C_A^{1*} = C^1(w_A^1, y_A^1)$

#### Firm B:

**Output** =  $y_B^1$

**Input price** =  $w_B^1$

**Cost** =  $C_B^1 = w_B^{1'} x_B^1$

**Minimum cost**  $C_B^{1*} = C^1(w_B^1, y_B^1)$

## Change in Cost Competitiveness over Time:

$$\frac{CCI_1}{CCI_0} = \frac{\frac{AC_B^1}{AC_A^1}}{\frac{AC_B^0}{AC_A^0}} = \frac{AC_B^1}{AC_B^0} \frac{AC_A^0}{AC_A^1}$$

***A* improves in cost competitiveness relative to *B* ,  
if  $AC_A$  fall faster than (or rises slower than)  $AC_B$ .**

For each firm  $j$  ( $j= A, B$ ) one can measure how its average cost changes over time by the (inter temporal) average cost index

$$ACI_j = \frac{AC_j^1}{AC_j^0}.$$

Change in Cost Competitiveness is

$$\frac{CCI_1}{CCI_0} = \frac{\frac{AC_B^1}{AC_B^0}}{\frac{AC_A^1}{AC_A^0}} = \frac{ACI_B}{ACI_A}.$$

To measure the change in the cost competitiveness of one firm relative to another we need to measure the average cost index of each firm and then compare the two.

$$\begin{aligned}
 ACI &= \frac{AC_1}{AC_0} = \frac{\frac{C_1}{y_1}}{\frac{C_0}{y_0}} \\
 &= \frac{\left(\frac{C_1}{C^1(w^1, y_1)}\right) \cdot \left(\frac{C^1(w^1, y_1)}{y_1}\right)}{\left(\frac{C_0}{C^0(w^0, y_0)}\right) \cdot \left(\frac{C^0(w^0, y_0)}{y_0}\right)} \\
 &= \frac{\left(\frac{C^0(w^0, y_0)}{C_0}\right) \cdot \left(\frac{C^1(w^1, y_1)}{y_1}\right)}{\left(\frac{C^1(w^1, y_1)}{C_1}\right) \cdot \left(\frac{C^0(w^0, y_0)}{y_0}\right)} \\
 &= \left[\frac{CE_0}{CE_1}\right] \cdot \left[\frac{\left(\frac{C^1(w^1, y_1)}{y_1}\right)}{\left(\frac{C^0(w^0, y_0)}{y_0}\right)}\right].
 \end{aligned}$$

$$\begin{aligned}
 & \left[ \frac{\left( \frac{C^1(w^1, y_1)}{y_1} \right)}{\left( \frac{C^0(w^0, y_0)}{y_0} \right)} \right] = \left[ \begin{array}{c} \frac{C^1(w^1, y^1)}{y^1} \\ \frac{C^1(w^1, y_1^*)}{y_1^*} \\ \frac{C^0(w^0, y^0)}{y^1} \\ \frac{C^0(w^0, y_0^*)}{y_0^*} \end{array} \right] \left[ \frac{C^1(w^1, y_1^*)}{y_1^*} \right] \\
 & \left[ \frac{C^1(w^1, y_1^*)}{y_1^*} \right] \\
 & = \left[ \frac{SE_1}{SE_0} \right] \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right] \cdot
 \end{aligned}$$

Now

$$\left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right] = \left[ \frac{C^1(w^1, y_1^*)}{C^1(w^0, y_1^*)} \right] \cdot \left[ \frac{\frac{C^1(w^0, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right] \cdot$$

Alternatively,

$$\left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right] = \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right] \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \right] \cdot$$

## Taking the geometric mean of the two

$$\left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right] = \left\{ \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right] \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \right] \cdot \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right] \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \right] \right\}^{\frac{1}{2}}$$

$$= \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right]^{\frac{1}{2}} \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \cdot \frac{\frac{C^1(w^0, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right]^{\frac{1}{2}}$$

**The first factor on the right hand side is a measure of ‘cost of production index’ and reflects input price change.**

Next consider the other term, which can be further broken up as

$$\begin{aligned} & \left[ \frac{C^1(w^1, y_1^*)}{y_1^*} \cdot \frac{C^1(w^0, y_1^*)}{y_1^*} \right]^{\frac{1}{2}} \\ & \frac{\frac{C^0(w^1, y_0^*)}{y_0^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \\ & = \left[ \frac{C^1(w^1, y_0^*)}{C^0(w^1, y_0^*)} \cdot \frac{C^1(w^0, y_1^*)}{C^0(w^0, y_1^*)} \right]^{\frac{1}{2}} \cdot \left[ \frac{C^1(w^1, y_1^*)}{y_1^*} \cdot \frac{C^0(w^0, y_1^*)}{y_1^*} \right]^{\frac{1}{2}} \cdot \left[ \frac{C^1(w^1, y_0^*)}{y_0^*} \cdot \frac{C^0(w^0, y_0^*)}{y_0^*} \right] \cdot \end{aligned}$$

Putting all the pieces together, a full blown decomposition of the average cost index is:

$$ACI = \frac{AC_1}{AC_0} = [CEC] \cdot [SEC] \cdot [IPC] \cdot [TC] \cdot [SBTC]$$

$$\left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right] = \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right] \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \right] \cdot$$

$$\left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right] = \left\{ \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right] \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \right] \cdot \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right] \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \right] \right\}^{\frac{1}{2}}$$

$$= \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \right]^{\frac{1}{2}} \cdot \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^1, y_0^*)}{y_0^*}} \cdot \frac{\frac{C^1(w^0, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right]^{\frac{1}{2}} \cdot$$

$$CEC = \frac{CE_0}{CE_1} = \frac{\left(\frac{C^0(w^0, y_0)}{C_0}\right)}{\left(\frac{C^1(w^1, y_1)}{C_1}\right)} \quad \text{is Cost Efficiency Change}$$

$$CEC < 1 \Rightarrow CE_1 > CE_0 \Rightarrow$$

the firm is more cost-competitive  
in period 1 relative to period 0.

**Cost Efficiency Change can be further broken up into  
changes in technical and allocative efficiencies**

$$SEC = \left[ \frac{SE_0}{SE_1} \right] = \frac{\frac{\frac{C^0(w^0, y_0^*)}{y_0^*}}{y_0^0}}{\frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{y_1^1}} \quad \text{is Scale Efficiency Change}$$

$$IPC = \left[ \frac{C^0(w^1, y_0^*)}{C^0(w^0, y_0^*)} \frac{C^1(w^1, y_1^*)}{C^1(w^0, y_1^*)} \right]^{\frac{1}{2}} \text{ is a measure of input price change.}$$

It is the geometric mean of the ratios of the ratio of the cost of producing the optimal output level in each period (0 and 1) at the two input price vectors ( $w^1$  and  $w^0$ ).

A value of  $IPC$  greater than 1 implies that input prices are higher in period 1 relative to period 0.

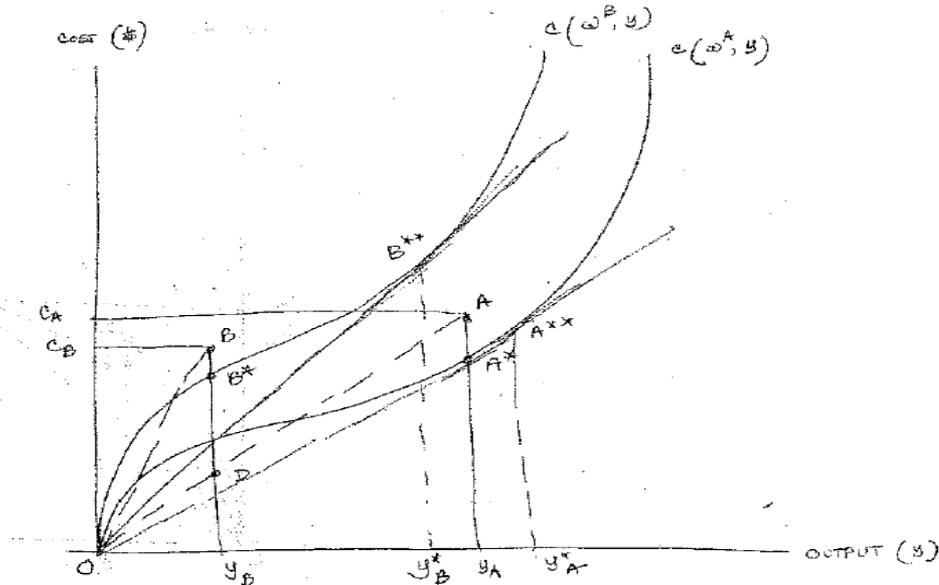
$$TC = \left[ \frac{C^1(w^1, y_0^*)}{C^0(w^1, y_0^*)} \frac{C^1(w^0, y_1^*)}{C^0(w^0, y_1^*)} \right]^{\frac{1}{2}} \text{ is a measure of Technical Change}$$

A value of  $TC$  less than unity implies that at the same input prices the cost of a given level of output is lower in period 1 than in period 0.

$$SBTC = \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^1(w^1, y_0^*)}{y_0^*}} \cdot \frac{\frac{C^0(w^0, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \right]^{\frac{1}{2}} = \left[ \frac{\frac{C^1(w^1, y_1^*)}{y_1^*}}{\frac{C^0(w^0, y_0^*)}{y_0^*}} \cdot \frac{\frac{C^0(w^0, y_1^*)}{y_1^*}}{\frac{C^1(w^1, y_0^*)}{y_0^*}} \right]^{\frac{1}{2}}$$

•

is the **Scale-Bias of Technical Change**.



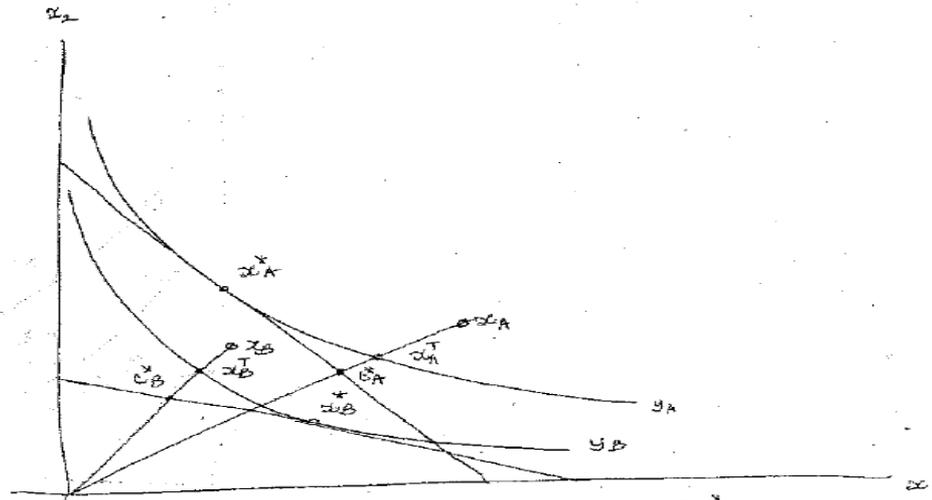
$$AC_A = \frac{Ay_A}{Oy_A} ; AC_B = \frac{By_B}{Oy_B}$$

$$CCI = \frac{AC_B}{AC_A} = \frac{By_B/Oy_B}{Ay_A/Oy_A} = \frac{By_B}{Ay_A} > 1$$

$$CE_A = \frac{A^*y_A}{Ay_A} ; CE_B = \frac{B^*y_B}{By_B}$$

$$SE_A = \frac{A^*y_A^*/Oy_A^*}{Ay_A/Oy_A} ; SE_B = \frac{B^*y_B^*/Oy_B^*}{By_B/Oy_B}$$

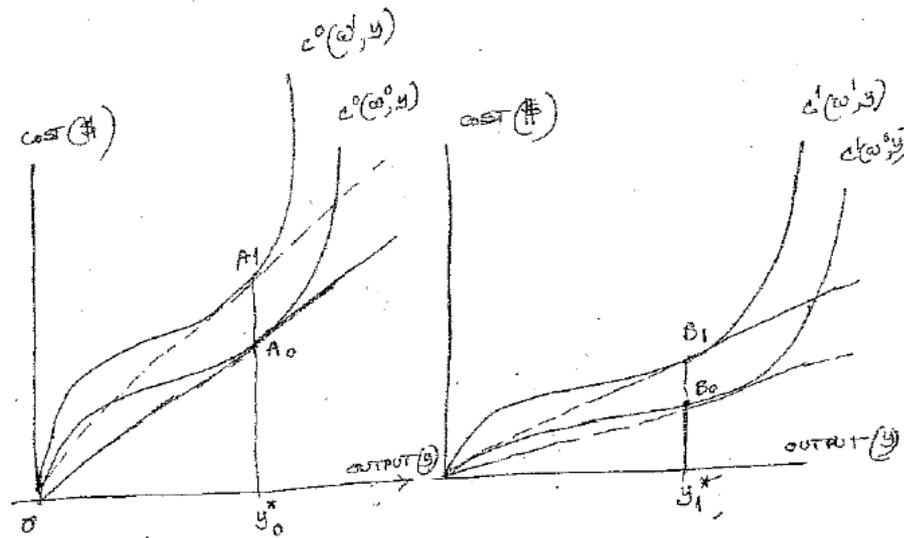
Figure 1



$$CE_A = \frac{Oz_A}{Oz_A} ; TE_A = \frac{Oz_A^T}{Oz_A} ; AE_A = \frac{Oz_A^*}{Oz_A^T}$$

$$CE_B = \frac{Oz_B}{Oz_B} ; TE_B = \frac{Oz_B^T}{Oz_B} ; AE_B = \frac{Oz_B^*}{Oz_B^T}$$

Figure 2



INPUT PRICE CHANGE

$$= \left[ \left( \frac{A_1 y_0^*}{A_0 y_0^*} \right) \cdot \left( \frac{B_1 y_0^*}{B_0 y_0^*} \right) \right]^{1/2}$$

Figure 3

## Santa Cruz Presentation

### **Contemporaneous Cost Competitiveness**

**Both firms face the same technology**

**But Input prices may differ**

$$\begin{aligned}
 CCI &= \frac{AC_B}{AC_A} = \frac{\frac{C_B}{y_B}}{\frac{C_A}{y_A}} \\
 &= \frac{\frac{C(w^A, y_A)}{C_A}}{\frac{C(w^B, y_B)}{C_B}} \cdot \left[ \frac{C(w^B, y_A^*)}{C(w^A, y_A^*)} \cdot \frac{C(w^B, y_B^*)}{C(w^A, y_B^*)} \right]^{\frac{1}{2}} \cdot \frac{\frac{AC(w^A, y_A^*)}{AC(w^A, y_A)}}{\frac{AC(w^B, y_B^*)}{AC(w^B, y_B)}} \cdot \left[ \frac{AC(w^A, y_B^*)}{AC(w^A, y_A^*)} \cdot \frac{AC(w^B, y_B^*)}{AC(w^B, y_A^*)} \right]^{\frac{1}{2}} \\
 &= \left[ \frac{CE_A}{CE_B} \right] [IPC] \left[ \frac{SE_A}{SE_B} \right] \cdot [n\_HOM]
 \end{aligned}$$

## **A 2-part Application to US Manufacturing Data**

**State level Data constructed from Census of Manufacturers**

**1) Cost Competitiveness of NC in 1992**

**2) Changes in Average Cost between 1997 and 2007**

## **Output:**

**Gross value of Production (Constant 1997 prices)**

## **Inputs:**

**Production Workers (persons)**

**Non-production workers (persons)**

**Energy (Btu)**

**Materials (Value in 1997 \$\$)**

**Capital: Gross Fixed Assets (in 1997\$\$)**

See Ray, Chen, and Mukherjee, *IJPE* 2008

year	name	CE	TE	AE	SE	IPC	N_HOMO	CCI
1992	CA	1.044	1.015	1.028	0.922	1.087	1	1.047
1992	IN	1.056	1.060	0.997	1.002	1.040	1	1.100
1992	MA	0.996	1.000	0.996	0.932	1.100	1	1.021
1992	MI	1.096	1.040	1.054	0.968	1.108	1	1.175
1992	NJ	1.027	1.000	1.027	0.932	1.087	1	1.041
1992	NY	0.976	1.000	0.976	0.918	1.086	1	0.973
1992	NC	1.000	1.000	1.000	1.000	1.000	1	1.000
1992	TX	1.148	1.019	1.126	0.969	1.027	1	1.143
1992	VA	0.976	1.000	0.976	0.976	1.011	1	0.964

## Decomposition of Cost Competitiveness Index of NC in 1992

## Summary of 1992 Results

- **NC was most cost competitive against MI (17.5%), TX (14.3%) and IN (10%)**
- **But NC was behind VA (by 3.6%) and NY (by 2.7%)**
- **NC had lower cost efficiency than NY and VA but also MA even though it was over competitive against MA.**
- **NC had a higher scale efficiency than all other states except IN.**
- **Overall input prices were lower in NC than all of the other states**

<b>name</b>	CE97/CE07	TE97/TE08	AE97/AE09	SE97/SE10
CA	0.871	1.004	0.867	0.861
IN	0.908	0.975	0.932	0.790
MA	0.850	1.000	0.850	0.885
MI	0.883	1.060	0.833	0.839
NJ	0.880	1.020	0.863	0.870
NY	0.888	0.979	0.907	0.792
NC	0.823	1.000	0.823	0.791
TX	0.818	0.956	0.856	0.895
VA	0.897	1.044	0.860	0.797

## Changes in Average Cost over Time 1997-2007

name	PC	TC	SBTC
CA	1.074	1.260	1.036
IN	1.063	1.291	1.036
MA	1.105	1.259	1.037
MI	1.080	1.285	1.039
NJ	1.109	1.276	1.039
NY	1.090	1.266	1.034
NC	1.101	1.265	1.032
TX	1.121	1.274	1.037
VA	1.101	1.275	1.036

### **Changes in Average Cost over Time**

## **Between 1997 and 2007**

- **All states experienced and increase in cost efficiency**
- **It was mainly due to an increase in allocative efficiency**
- **CA, MI, NJ, and VA saw decline in technical efficiency**
- **While all states improved in scale efficiency, IN, NY, and NC improved most**
- **Input prices increased in all states: by over 10% in MA, NJ, NC, TX, and VA and by 6.3% to 9% in other states**
- **There was an autonomous upward shift in the cost function in 2007 relative to 1997.**
- **This implies technical regress (may be due to regulatory changes).**

**Thank you!**