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An Explanation of Virtual Population Analysis

by

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Virtual population analysis (VPA) is a technique employed to calculate fishing mortality rates and stock sizes (abundance) of a year-class (the fish hatched in a given year) at successive ages, given the known or assumed natural mortality rate and catch (in numbers) at each age and given the known or assumed fishing mortality rate at the oldest age. In order to describe the technique, it is first necessary to define some basic terms and equations relating to mortality rates, stock size, and catch.

If the abundance (N) of a year-class is known at the beginning of two consecutive years, N_i and N_{i+1}, the fraction surviving from year i to year i+1 is

$$\frac{N_{i+1}}{N_i} = S_i \tag{1}$$

where S_i is the annual survival rate. It then follows that

$$S_i = 1 - A_i \tag{2}$$

where A_i is the annual mortality rate. In working with mortality rates it is most convenient to deal with instantaneous rates. Instantaneous total mortality is defined as

$$Z_i = -\log_e (1-A_i) \tag{3}$$

or

$$1 - A_i = e^{-Z_i} = S_i \tag{4}$$

Therefore

$$\frac{N_{i+1}}{N_i} = e^{-Z_i} \tag{5}$$

There are two major causes of death or removal of fish from a stock: (1) fishing and (2) natural death or all causes except fishing (e.g. predation, disease, senility, etc.). These two types of mortality are also expressed as instantaneous rates: (1) F or instantaneous fishing mortality and (2) M or instantaneous natural mortality. The sum of these is instantaneous total mortality, or

$$F + M = Z \tag{6}$$

If N_i represents the abundance of a given year-class of fish at the beginning of year i, then the total number of deaths (fishing + natural) during the year is the product of N_i and the annual mortality rate or $N_i A_i$; substituting $1-e^{-Z_i}$ for A_i from equation (4) gives $N_i(1-e^{-Z_i})$. The number of deaths due only to fishing (catch or C_i) can be calculated by applying $\frac{F_i}{Z_i}$ (the ratio between fishing mortality and total mortality) to the above expression representing total deaths. This gives what is often known as the catch equation where

$$C_i = N_i \frac{F_i}{Z_i} (1-e^{-Z_i}) \tag{7}$$

The expression $\frac{F_i}{Z_i} (1-e^{-Z_i})$ is referred to as the rate of exploitation and represents the proportion (by number) of the fish in a year-class alive at the beginning of year i which are caught during year i.

The abundance of a year-class at the beginning of year i+1 can be expressed as a proportion (r) of the catch during year i,

$$r_i = \frac{N_{i+1}}{C_i} \tag{8}$$

Based on equations (5) and (7), equation (8) can be written as

$$r_i = \frac{N_{i+1}}{C_i} = \frac{N_i e^{-Z_i}}{N_i \frac{F_i}{Z_i}(1-e^{-Z_i})} = \frac{Z_i e^{-Z_i}}{F_i(1-e^{-Z_i})} \qquad (9)$$

Therefore, if N_{i+1} and C_i are both known, F_i can be solved by iteration which is time-consuming unless done by computer. In other words, by trial-and-error process the correct value for F_i is determined such that the expression $\frac{Z_i e^{-Z_i}}{F_i(1-e^{-Z_i})}$ equals the ratio $\frac{N_{i+1}}{C_i}$

The computations involved in a VPA can be illustrated by means of an example. The following table presents a hypothetical year-class consisting of 10,000 fish at age 1 which is subjected to fishing and natural mortality over a 12-year life span. The year-class size (N), the catch (C), the instantaneous fishing (F) and natural (M) mortality rates, and the number lost due to natural death are listed for each age.

Age	N	C	F	M	Natural Death
1	10,000	100	.012	.4	3,278
2	6,622	600	.116	.4	2,069
3	3,953	1,000	.361	.4	1,107
4	1,846	500	.392	.4	510
5	836	200	.338	.4	236
6	400	100	.357	.4	113
7	187	50	.385	.4	52
8	85	25	.430	.4	23
9	37	10	.388	.4	10
10	17	5	.435	.4	5
11	7	2	.394	.4	2
12	3	1	.446	.4	2
		2,593			7,407

Suppose that a scientist wishes to perform a VPA to determine fishing mortality and stock size at each age. Assume that (1) the catch at each age is known, (2) M has been estimated as 0.4 from previous research, and (3) F at age 12 has been estimated by some method or procedure to be 0.5. The first step in the VPA computations is to determine the stock size at age 12 or N_{12}, F_{12}. Since C_{12}, F_{12}, and Z_{12} (which is F_{12} + M) are known or estimated, N_{12} is solved for by means of the catch equation (7). The value for N_{12} is calculated to be 3.

The second step is to determine the fishing mortality at age 11 or F_{11}. Since N_{12} (3) and C_{11} (2) are known, F_{11} is solved for by computer iteration of equation (9). In this way F_{11} is determined to be 0.424. Using the catch equation (7), the value for N_{11} is determined to be 7. The remaining values of F and N for ages 10 through 1 are determined by the repeated use of equations (7) and (9). The following table shows the values of F and N at each age calculated by VPA.

Age	C	F	N
1	100	.012	9,969
2	600	.116	6,606
3	1,000	.363	3,942
4	500	.394	1,839
5	200	.341	831
6	100	.360	396
7	50	.391	185
8	25	.440	84
9	10	.400	36
10	5	.457	16
11	2	.424	7
12	1	.500	3

Comparison of the F and N values above with the actual F and N values in the first table indicates very little difference. The F values from the VPA are slightly larger than the actual F values, whereas the N values from the VPA are slightly smaller than the actual N values. This occurred because the estimated value for F_{12} of 0.5 was slightly greater than the actual value of 0.446. If the estimated F_{12} value had been less than 0.446, the calculated F and N values would have been smaller and larger, respectively, than the actual values. The greater the error in the estimated F_{12} value, the greater the error in the calculated F and N values. However, the error is greatest for the first few ages after the starting point and then diminishes rapidly because the computed F values converge asymptotically to their actual values.

The above example describes the application of VPA to the catch data for a single year-class. Application of the technique to all year-classes represented in the data base for the stock in question will provide estimates of fishing mortality and stock size at all ages in each calendar year represented. Stock sizes by age in each year so determined can be summed to give total stock size in each year. Fishing mortality by age in each year will indicate the mortality pattern by age and permit the calculation of an average mortality rate for each year.

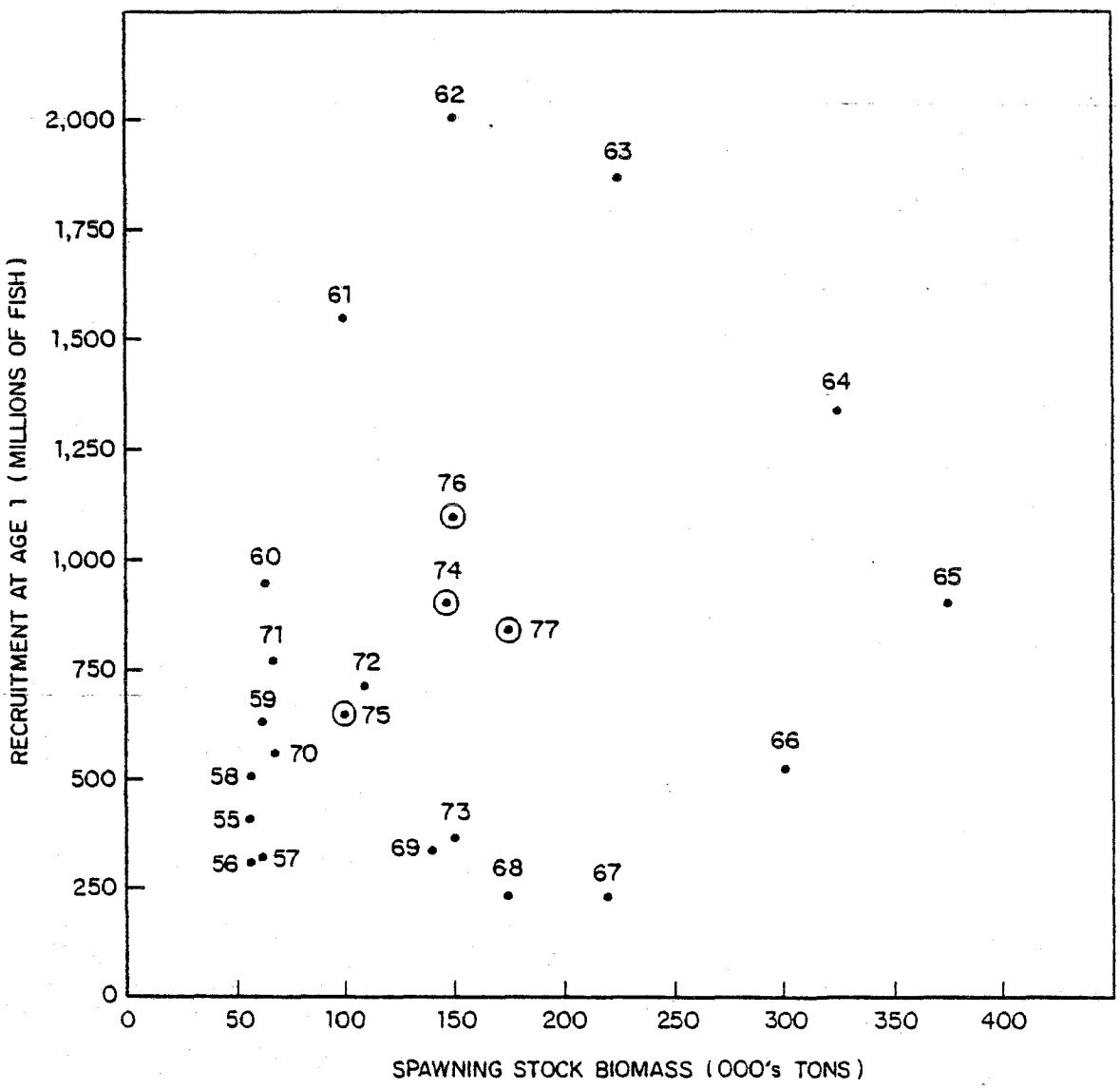

Figure 6. Relationship between spawning stock biomass (age 2+) and recruitment (expressed as abundance at age 1) for the Southern New England - Middle Atlantic silver hake stock during 1955-77. Open circles indicate estimated year-class sizes.