Appendix B1. Exploration of the Statistical Catch-at-Age

Data and Methodology
The algebraic details of the methods used for the SCAA assessments and BRP estimation are set out in Appendix B2.

The following changes have been made from "2011 - new data" assessment with which the bridge-building exercise culminates to provide the provisional new Reference Case assessment “RCp”:

10. Survey season: spring and autumn instead of begin and mid-year (equation B2.9).
11. Survey variance: use input CV's and estimate additional variance (equation B2.16), instead of estimate year-independent variance.
12. \( \phi \) estimated instead of fixed at 0.2.
13. \( \mu_{\text{spawn}} \approx 0.25 \) instead of 0.1667 (equation B2.6).
14. Use age-dependent \( \sigma_a \) for CAA (equations B2.18 and B2.21).
15. Flat commercial selectivity from age 6.

The first six of these changes are either necessitated by changes to or more accurate representation of input information, together with advances made since GARM III in the assessment methodology applied to other stocks in the region such as Gulf of Maine cod (see e.g. Butterworth and Rademeyer 2012). The necessity for change 6 in the case of white hake was confirmed through the use of AIC. Changes 7 and 8 eventuated from specific analyses for the preliminary white hake data. Regarding 7, freeing the parameter concerned resulted in only a very weak dome in the commercial selectivity vector, and little improvement of the likelihood or changes in key results compared to keeping selectivity flat at larger ages, so it was set to be flat for RCp. Inspection of proportions-at-age residuals suggested a systematic pattern change for the commercial catch proportions-at-age in the mid-1990s. Katherine Sosebee suggested two specific possibilities for the time of this change based on other information; a change from 1997 to 1998 was selected for distinguishing two commercial selectivity blocks based on a better AIC (where this criterion also clearly justified the split from the previous single block).

The list of sensitivities to RCp that are presented in this paper is given in Appendix Table B1.1.

Results
Appendix Table B1.2 lists estimates of primary parameters and management-related quantities for Georges' Bank/Gulf of Maine white hake for RCp and a series of sensitivities. Estimates of BRPs and current stock status estimates are summarized in Appendix Table B1.3. Additional runs, including the final run that was compared to the ASAP model are summarized in Appendix Table B1.4.

Appendix Figure B1.1 gives results for the RCp, while Appendix Figure B1.2 plots its fit to survey and commercial data. Appendix Figure B1.3 compares spawning biomass and recruitment trajectories for RCp and the different sensitivities. Appendix Figure B1.4 compares the stock-recruitment curves for RCp (Ricker), sensitivity 2a (Beverton-Holt) and sensitivity 2b (modified Ricker, with \( \gamma \) estimated). The commercial and survey selectivities for RCp and the sensitivities related to selectivities (4a/b/c/d) are plotted in Appendix Figure B1.5. Bubble plots of CAA residuals are compared for RCp, 4a (flat survey selectivity), 6a (sqrt(p)) and 6b (sqrt(p), flat survey selectivity). The fits to the survey and commercial CAA and CAL data for sensitivity 8c, for which CAA from pooled ALKs are excluded and replaced by CAL, are shown in Appendix Figure B1.6. The fits to the survey biomass indices for sensitivity 9a, in which the RV Albatross/FRV Henry B. Bigelow calibration factor is estimated, are plotted in Appendix Figure B1.7.

Discussion

1) The fits to the data do not suggest \( M \) values greater than 0.2. (Sensitivity 1)
2) The Ricker stock-recruitment form is favoured over Beverton-Holt, with the data suggesting a sharper peak than the standard Ricker form, though the evidence for preference in terms of improvements to the likelihood is not strong. (Sensitivity 2)

3) Fitting to aggregate abundance indices in terms of numbers, rather than biomass, results in higher current and pristine spawning biomass estimates, but current stock status relative to the MSY spawning biomass level is not greatly affected. If only the spring NEFSC survey data are used, this status is improved, with the reverse result if only the autumn survey data are used. (Sensitivity 3)

4) Investigation of alternative assumptions for selectivity functions show strong AIC support for a difference in the slopes of commercial and survey selectivities-at-age above age 6, with a preference for a near-flat commercial selectivity and strongly domed survey selectivities. The alternative sqrt(p) formulation for the distribution of the proportions-at-age residuals finds this same result, and suggests slightly improved current resource status relative to the MSY spawning biomass level than does the adjusted log-normal of RCp. Shifting the pre-1982 commercial selectivity towards a relatively larger catch of smaller hake has little impact on results. (Sensitivities 4 and 6)

5) When starting the assessment in 1963, the parameter which determines the initial age structure is poorly estimated, but this doesn’t impact seriously on the estimates of biological reference points in terms of precision, with starting in 1950 instead also making little difference (note results falling well within CIs for the 1963 start in early years in Fig. 3a). In contrast, for a start in 1982, although phi becomes estimable with reasonable precision, the stock-recruitment relationship cannot be reasonably estimated. (Sensitivity 5)

6) Removable of an internally estimated stock-recruitment relationship results, through differences in the related shrinkage of recent estimates of recruitment, in lower estimates of current abundance. (Sensitivity 7)

7) Without inclusion of catch proportions-at-age data for years without direct ageing through use of an average ALK, the precision of the estimates of many quantities deteriorates substantially. However fitting to catch-at-length data for those years provides near unchanged results in terms of both these values and their precision. (Sensitivity 8).

8) Refining the RV Albatross/FRV Henry B. Bigelow calibration factor within the assessment leads to a slightly improved estimate of current stock status. The estimate of this factor decreases from 2.235 to 2.096, with an improvement in the associated standard error from 0.173 to 0.155. (Sensitivity 9)

9) The RCp assessment and a number of key sensitivities all suggest that at present the stock is not overfished and that overfishing is not occurring. Estimates of current status and of catches under 0.75 F_{MSY} are rather more optimistic when based on fitted stock-recruitment curves than on F40% MSY proxies. For the latter, starting the assessment in 1963 yields slightly more positive results than starting it in 1982. (Appendix Table B1.3)
Appendix Table B1.1: List of the sensitivities run. After each sub-heading, the RCp specifications are given in parenthesis.

1. Natural mortality (RCp: $M=0.2$)
   1a. $M=0.4$
   1b. $M$ incr: $M$ increasing linearly from 0.2 at age 5 to 0.4 at age 9
2. Stock-recruitment curve (RCp: Ricker)
   2a BH: Beverton-Holt stock-recruitment curve
   2b $\gamma$ estimated: from the modified Ricker, eqn B2.4
3. Survey data (RCp: Fit to biomass, both surveys)
   3a Fit to numbers: for the survey indices
   3b Fit to Spring survey only: for both the index and CAA data
   3c Fit to Autumn survey only: for both the index and CAA data
4. Selectivities (RCp: flat comm. From age 6, domed survey)
   4a Flat survey selectivity: from age 6
   4b Pre-1982 comm sel shifted: shifted one year to the left
   4c Flat survey sel, domed comm. Sel: flat from age 6 for survey, free for commercial
   4d Domed survey and comm. Sel
5. Start year (RCp: start in 1963)
   5a Start in 1982
   5b Start in 1950
6. CAA error formulation (RCp: adjusted log-normal)
   6a $\sqrt{p}$
   6b $\sqrt{p}$, flat survey selectivity
7. No internal stock-recruitment (RCp: internal stock-recruit)
   7a no SR
   7b no SR, start 1982
8. Excluding CAA from pooled ALK (RCp: include CAA from pooled ALK)
   8a Survey CAL for yrs with pooled ALK
   8b Surv and comm CAL for yrs with pooled ALK
   8c Exclude CAA from pooled ALK: not fitting to any CAL
9. Calibration refinement (RCp: calibration refinement not included)
   9a Bigelow calibration: $\Delta ln q$ estimated (equation B2.33)
Appendix Table B1.2a: Results for RCp and some sensitivities. Mass units are ‘000 tons.

<table>
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<tr>
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<th>RCp</th>
<th>1a</th>
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<th>2a</th>
<th>2b</th>
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<td>Fit to Numbers</td>
<td>Fit to Spring survey only</td>
<td>Fit to Autumn survey only</td>
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<td>1-InL: overall</td>
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<td>-301.6</td>
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\(h\) 1.21 (0.14) 0.62 (0.15) 0.74 (0.15) 0.78 (0.09) 1.26 (0.13) 0.81 (0.14) 1.30 (0.15) 1.24 (0.15)

\(\gamma\) 1.00 - 1.00 - 1.00 - 1.00 - 2.11 (0.50) 1.00 - 1.00 - 1.00 -

\(\theta\) 0.57 (0.29) 0.57 (0.21) 0.56 (0.19) 0.28 (0.34) 0.77 (0.17) 0.25 (0.29) 0.77 (0.19) 0.52 (0.28)

\(\phi\) 0.01 (4.07) 0.00 (1000) 0.00 (1000) 0.02 (1.65) 0.00 (1000) 0.03 (4.07) 0.00 (1000) 0.02 (1.81)

\(K^{SP}\) 69.13 (0.14) 68.91 (0.19) 66.39 (0.17) 128.17 (0.20) 55.06 (0.17) 120.65 (0.14) 71.01 (0.14) 64.82 (0.15)

\(B^{SP}_{2011}\) 25.34 (0.17) 37.17 (0.18) 32.38 (0.18) 24.77 (0.17) 25.25 (0.18) 29.78 (0.17) 33.99 (0.23) 22.45 (0.19)

\(B^{SP}_{2011}/K^{SP}\) 0.37 (0.21) 0.54 (0.24) 0.49 (0.22) 0.19 (0.26) 0.46 (0.21) 0.25 (0.21) 0.48 (0.26) 0.35 (0.23)

\(B^{SP}_{MSY}/K^{SP}\) 30.43 (0.10) 32.35 (0.13) 31.57 (0.12) 42.98 (0.16) 29.38 (0.13) 39.44 (0.10) 31.05 (0.11) 28.53 (0.10)

\(MSY^{SP}\) 0.44 (0.11) 0.47 (0.16) 0.48 (0.19) 0.34 (0.07) 0.53 (0.24) 0.33 (0.11) 0.44 (0.12) 0.44 (0.11)

\(B^{SP}_{2011}/B^{SP}_{MSY}\) 0.83 (0.18) 1.15 (0.18) 1.03 (0.18) 0.58 (0.23) 0.86 (0.20) 0.76 (0.18) 1.09 (0.22) 0.79 (0.19)

\(MSY\) 7.75 (0.10) 8.37 (0.13) 8.39 (0.12) 7.82 (0.15) 8.57 (0.13) 7.60 (0.10) 8.44 (0.10) 7.41 (0.10)

\(F_{MSY}\) 0.30 - 0.41 - 0.35 - 0.21 - 0.35 - 0.22 - 0.33 - 0.31 -

\(spring\_q\) 1.16 (0.06) 0.54 (0.07) 0.86 (0.07) 1.16 (0.06) 1.16 (0.06) 1.06 (0.06) 1.10 (0.06) -

\(autumn\_q\) 1.96 (0.05) 0.97 (0.07) 1.42 (0.07) 1.97 (0.05) 1.97 (0.05) 1.71 (0.05) - 2.04 (0.05)

\(spring\_\sigma_{Add}\) 0.16 (0.32) 0.17 (0.32) 0.16 (0.32) 0.16 (0.32) 0.16 (0.32) 0.13 (0.31) 0.20 (0.29) -

\(autumn\_\sigma_{Add}\) 0.06 (0.48) 0.10 (0.40) 0.09 (0.41) 0.05 (0.49) 0.05 (0.49) 0.14 (0.30) - 0.07 (0.33)
Appendix Table B.2b: Results for RCp and some sensitivities. Mass units are ‘000 tons.

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<th>B^SP^2012/K^SP</th>
<th>B^SP^MSY</th>
<th>MSYL^SP</th>
<th>B^SP^2012/B^SP^MSY</th>
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<th>F^MSY</th>
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Appendix Table B1.2c: Results for RCp and some sensitivities. Note that for 7a, the BRP are estimated externally to the assessment (see Appendix B2, section B2.5). For sensitivity 9a (Bigelow calibration), the first two survey q’s (and associated CVs) are for the *Albatross*, followed by those for the *Bigelow*. Mass units are ‘000 tons.

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<th>6a</th>
<th>6b</th>
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<th>7b</th>
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Calibration Ratio

|                | 2.09 | 2.09         | 2.24         | 2.24  | 2.24 | 2.24  | 2.24  | 2.24  | 2.24        | 2.24    | 2.24        | 2.24    | 2.24        |

$\beta_{2011}/k^0$
Appendix Figure B1.1: Results for the RCp Georges Bank/Gulf of Maine white hake assessment.
Appendix Figure B1.2: Fit of RCp to the survey and commercial data
Appendix Figure B1.3a: Spawning biomass and recruitment trajectories for RCp and some sensitivities. The 95% CIs shown in the bottom left plot are for RCp.
Appendix Figure B1.3b: Spawning biomass and recruitment trajectories for RCp and some sensitivities.
Appendix Figure B1.4: Stock-recruitment curve and estimated recruitment for RCp (full line and solid dots) and 2a (Beverton-Holt) (dashed line and crosses) for the left-hand plot and 2b (γ estimated) (dashed line and crosses) for the right-hand plot. Note that that N1 values for year y are associated with spawning biomass values for the previous year.

Appendix Figure B1.5: Commercial and survey selectivities for RCp and some sensitivities.
Appendix Figure B1.6: CAA standardised residuals for RCp and some sensitivities.
Appendix Figure B1.7: Fit to CAA and CAL for sensitivity 8c.
Appendix Figure B1.8: Fit to NEFSC surveys adjusted for the calibration refinement. Open circles are the surveys with the existing calibration factor.
Appendix Figure B1.9a: Spawning biomass and recruitment trajectories for EvenNewerRCp and some sensitivities.
Appendix Figure B1.9b: Spawning biomass and recruitment trajectories for EvenNewerRCp and some sensitivities and a version of the ASAP.
Appendix Figure B1.10. Spawner-recruit plots from RCNewer to BH and noSR
Appendix Figure B1.11: Results for the RCpEvenNewer Georges Bank/Gulf of Maine white hake assessment.
Appendix Figure B1.12a: Fit of RCpEvenNewer to the survey and commercial data
Appendix Figure B1.12b: Fit of RCpEvenNewer to the survey and commercial data
Appendix B2

Algebraic details of the Statistical Catch-at-Age Model

The text following sets out the equations and other general specifications of the Statistical Catch-at-Age (SCAA) assessment model applied to white hake, followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is applied to minimize the total negative log-likelihood function to estimate parameter values (the package AD Model Builder™, Otter Research, Ltd is used for this purpose).

Where options are provided under a particular section, the section concludes with a statement in **bold** as to which option was selected for the provisional Reference Case (RCp) run selected.

B2.1. Population dynamics

B2.1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

\[ N_{y+1,1} = R_y \] (B2.1)

\[ N_{y+1,a+1} = N_{y,a} e^{-Z_{y,a}} \quad \text{for } 1 \leq a \leq m - 2 \] (B2.2)

\[ N_{y+1,m} = N_{y,m-1} e^{-Z_{y,m-1}} + N_{y,m} e^{-Z_{y,m}} \] (B2.3)

where

- \( N_{y,a} \) is the number of fish of age \( a \) at the start of year \( y \),
- \( R_y \) is the recruitment (number of 1-year-old fish) at the start of year \( y \),
- \( m \) is the maximum age considered (taken to be a plus-group),
- \( Z_{y,a} = F_y S_{y,a} + M_a \) is the total mortality in year \( y \) on fish of age \( a \), where
  - \( M_a \) denotes the natural mortality rate for fish of age \( a \),
  - \( F_y \) is the fishing mortality of a fully selected age class in year \( y \), and
  - \( S_{y,a} \) is the commercial selectivity at age \( a \) for year \( y \).

B2.1.2. Recruitment

The number of recruits (i.e. new 1-year olds) at the start of year \( y \) is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) by either a modified Ricker or a Beverton-Holt stock-recruitment relationship, allowing for annual fluctuation about the deterministic relationship.

For the modified Ricker:

\[ R_y = \alpha B_{y-1}^{vp} \exp \left[ -\beta (B_{y-1}^{vp})^{1/4} \right] \left( e^{-\left( \sigma_h \right)^{2}/2} \right) \] (B2.4)
and for the (standard) Beverton-Holt:

\[ R_y = \frac{\alpha B_{y-1}^{sp}}{\beta + B_{y-1}^{sp}} e^{\frac{\zeta_y - (\sigma_y^2/2)}{\sigma_y^2}} \]  \hspace{1cm} (B2.5)

where

\( \alpha, \beta, \) and \( \gamma \) are spawning biomass-recruitment relationship parameters,

\( \zeta_y \) reflects fluctuation about the expected recruitment for year \( y \), which is assumed to be normally distributed with standard deviation \( \sigma_B \) (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.

\( B_{y-1}^{sp} \) is the spawning biomass at the start of year \( y \), computed as:

\[ B_{y-1}^{sp} = \sum_{a=1}^{m} f_a w_{y,a}^{\text{mint}} N_{y,a} e^{-Z_{y,a} \mu_{\text{spawn}}} \]  \hspace{1cm} (B2.6)

because spawning for the cod stock under consideration is taken to occur three months (\( \mu_{\text{spawn}} = 0.25 \)) after the start of the year and some mortality has therefore occurred,

where

\( w_{y,a}^{\text{mint}} \) is the mass of fish of age \( a \) during spawning, and

\( f_a \) is the proportion of fish of age \( a \) that are mature.

For RCp, the modified Ricker, with \( \gamma \) fixed to 1, has been used, i.e. the classical Ricker function.

B2.1.3. Total catch and catches-at-age

The total catch by mass in year \( y \) is given by:

\[ C_y = \sum_{a=1}^{m} w_{y,a}^{\text{m}id} C_{y,a} = \sum_{a=1}^{m} w_{y,a}^{\text{m}id} N_{y,a} S_{y,a} F_y \left( 1 - e^{-Z_{y,a}} \right) \]  \hspace{1cm} (B2.7)

where

\( w_{y,a}^{\text{m}id} \) denotes the mass of fish of age \( a \) landed in year \( y \),

\( C_{y,a} \) is the catch-at-age, i.e. the number of fish of age \( a \), caught in year \( y \).

The model estimate of survey index is computed as:

\[ B_{y}^{\text{surv}} = \sum_{a=1}^{m} w_{y,a}^{\text{surv}} S_{a} N_{y,a} e^{-Z_{y,a} \mu_{\text{spawn}}/12} \]  \hspace{1cm} (B2.8)

for biomass indices and
\[ N_{y, a}^{\text{surv}} = \sum_{a=1}^{m} S_a^{\text{surv}} N_{y,a} e^{-Z_{a,T}^{\text{surv}}/12} \quad (B2.9) \]

for numbers indices

where

- \( S_a^{\text{surv}} \) is the survey selectivity for age \( a \), which is taken to be year-independent.
- \( T^{\text{surv}} \) is the season in which the survey is taking place (\( T^{\text{surv}} = 3 \) for spring surveys and \( T^{\text{surv}} = 9 \) for fall surveys), and
- \( w_{y,a}^{\text{surv}} \) denotes the mass of fish of age \( a \) from survey \( \text{surv} \) year, taken as \( w_{y,a}^{\text{surv}} \) for the spring survey and \( w_{y,a}^{\text{mid}} \) for the autumn survey.

**RCp is fitted to biomass indices.**

**B2.1.4. Initial conditions**

As the first year for which data (even annual catch data) are available for the white hake stock considered clearly does not correspond to the first year of (appreciable) exploitation, one cannot necessarily make the conventional assumption in the application of SCAA’s that this initial year reflects a population (and its age-structure) at pre-exploitation equilibrium. For the first year (\( y_0 \)) considered in the model therefore, the stock is assumed to be at a fraction (\( \theta \)) of its pre-exploitation biomass, i.e.:

\[ B_{y_0}^p = \theta \cdot K^p \quad (B2.10) \]

with the starting age structure:

\[ N_{y_0,a} = R_{\text{start}} N_{\text{start},a} \quad \text{for} \quad 1 \leq a \leq m \quad (B2.11) \]

where

\[ N_{\text{start},1} = 1 \quad (B2.12) \]

\[ N_{\text{start},a} = N_{\text{start},a-1} e^{-M_{a-1} (1 - \phi S_{a-1})} \quad \text{for} \quad 2 \leq a \leq m - 1 \quad (B2.13) \]

\[ N_{\text{start},m} = N_{\text{start},m-1} e^{-M_{m-1} (1 - \phi S_{m-1})} / (1 - e^{-M_m (1 - \phi S_m)}) \quad (B2.14) \]

where \( \phi \) characterises the average fishing proportion over the years immediately preceding \( y_0 \).

For RCp, \( \theta \) and \( \phi \) are estimated directly in the model fitting procedure.

**B2.2. The (penalised) likelihood function**

The model can be fit to (a subset of) survey abundance indices, and commercial and survey catch-at-age and catch-at-length data to estimate model parameters (which may include residuals about the stock-recruitment function, facilitated through the incorporation of a penalty function described below). Contributions by each of these to the negative of the (penalised) log-likelihood (\( -\ell n L \)) are as follows.
B2.2.1. Survey abundance data

The likelihood is calculated assuming that a survey biomass index is log normally distributed about its expected value:

\[
I_{y}^{\text{surv}} = \hat{I}_{y}^{\text{surv}} \exp(\varepsilon_{y}^{\text{surv}}) \quad \text{or} \quad \varepsilon_{y}^{\text{surv}} = \ln(I_{y}^{\text{surv}}) - \ln(\hat{I}_{y}^{\text{surv}}) \quad (B2.15)
\]

where

\(I_{y}^{\text{surv}}\) is the survey index for survey \(\text{surv}\) in year \(y\),

\(\hat{I}_{y}^{\text{surv}} = \hat{q}^{\text{surv}} \hat{B}_{y}^{\text{surv}}\) is the corresponding model estimate, where

\(\hat{q}^{\text{surv}}\) is the constant of proportionality (catchability) for the survey biomass series \(\text{surv}\), and

\(\varepsilon_{y}^{\text{surv}}\) from \(N(0, \sigma_{y}^{2})\).

The contribution of the survey biomass data to the negative of the log-likelihood function (after removal of constants) is then given by:

\[
-\ln L^{\text{survey}} = \sum_{\text{surv}} \sum_{y} \left\{ \ln \left( \sqrt{\left(\sigma_{y}^{\text{surv}}\right)^2 + \left(\sigma_{\text{Add}}^{\text{surv}}\right)^2} \right) + \left(\varepsilon_{y}^{\text{surv}}\right)^2 / 2 \left(\sigma_{y}^{\text{surv}}\right)^2 + \left(\sigma_{\text{Add}}^{\text{surv}}\right)^2 \right\} \quad (B2.16)
\]

where

\(\sigma_{y}^{\text{surv}}\) is the standard deviation of the residuals for the logarithm of index \(i\) in year \(y\) (which are input), and

\(\sigma_{\text{Add}}^{\text{surv}}\) is the square root of the additional variance for survey biomass series \(\text{surv}\), which is estimated in the model fitting procedure, with an upper bound of 0.5.

The catchability coefficient \(q^{\text{surv}}\) for survey biomass index \(\text{surv}\) is estimated by its maximum likelihood value:

\[
\ln q^{\text{surv}} = 1/n_{\text{surv}} \sum_{y} \left( \ln I_{y}^{\text{surv}} - \ln \hat{B}_{y}^{\text{surv}} \right) \quad (B2.17)
\]

B2.2.3. Commercial catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function under the assumption of an “adjusted” lognormal error distribution is given by:

\[
-\ln L^{\text{CAA}} = \sum_{y} \sum_{a} \left\{ \ln \left( \sigma_{a}^{\text{com}} / \sqrt{p_{y,a}} \right) + p_{y,a} \left( \ln p_{y,a} - \ln \hat{p}_{y,a} \right)^2 / 2 \left(\sigma_{a}^{\text{com}}\right)^2 \right\} \quad (B2.18)
\]

where

\(p_{y,a} = C_{y,a} / \sum_{a} C_{y,a}\) is the observed proportion of fish caught in year \(y\) that are of age \(a\),

\(\hat{p}_{y,a} = \hat{C}_{y,a} / \sum_{a} \hat{C}_{y,a}\) is the model-predicted proportion of fish caught in year \(y\) that are of age \(a\),
\[ \hat{C}_{y,a} = N_{y,a} S_{y,a} F_y \left( 1 - e^{-Z_{y,a}} \right) / Z_{y,a} \]  
(B2.19)

and

\[ \sigma_{a,com} \]  
is the standard deviation associated with the catch-at-age data, which is estimated in the fitting procedure by:

\[ \hat{\sigma}_{a,com} = \sqrt{\sum_y p_{y,a} \left( \ell n p_{y,a} - \ell n \hat{p}_{y,a} \right)^2 / \sum_y 1} \]  
(B2.20)

Commercial catches-at-age are incorporated in the likelihood function using equation (A1.18), for which the summation over age \( a \) is taken from age \( a_{\text{minus}} \) (considered as a minus group) to \( a_{\text{plus}} \) (a plus group).

In addition to this “adjusted” lognormal error distribution, some computations use an alternative “sqrt(p)” formulation, for which equation A1.18 is modified to:

\[ -\ell n L^{\text{CAA}} = \sum_y \sum_a \left[ \ell n(\sigma_{a,com}) + \left( \sqrt{p_{y,a}} - \sqrt{\hat{p}_{y,a}} \right)^2 / 2(\sigma_{a,com})^2 \right] \]  
(B2.21)

and equation A1.20 is adjusted similarly:

\[ \hat{\sigma}_{a,com} = \sqrt{\sum_y \left( \sqrt{p_{y,a}} - \sqrt{\hat{p}_{y,a}} \right)^2 / \sum_y 1} \]  
(B2.22)

This formulation mimics a multinomial form for the error distribution by forcing a near-equivalent variance-mean relationship for the error distributions.

B2.2.4. Survey catches-at-age

The survey catches-at-age are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-age, assuming an “adjusted” lognormal error distribution (equation (A1.18)) where:

\[ p_{y,a}^{\text{surv}} = C_{y,a}^{\text{surv}} \sum a' C_{y,a'}^{\text{surv}} \]  
is the observed proportion of fish of age \( a \) in year \( y \) for survey \( \text{surv} \),

\[ \hat{p}_{y,a}^{\text{surv}} \]  
is the expected proportion of fish of age \( a \) in year \( y \) in the survey \( \text{surv} \), given by:

\[ \hat{p}_{y,a}^{\text{surv}} = S_{a} N_{y,a} e^{-Z_{y,a} T^{\text{surv}} / 2} \sum_{a'=1}^{m} S_{a'} N_{y,a'} e^{-Z_{y,a'} T^{\text{surv}} / 2}. \]  
(B2.23)

RCp uses the “adjusted log-normal” formulation for the error distribution of the commercial catch proportions-at-age and survey catch proportions-at-age.

B2.2.5. Survey catches-at-length

In some runs, catches-at-length are also incorporated in the likelihood function. These data are incorporated in the similar manner as the catches-at-age. When the model is fit to catches-at-length, the predicted catches-at-age are converted to catches-at-length:

\[ \hat{p}_{y,a}^{\text{surv}} = \sum a' p_{y,a'}^{\text{surv}} A_{a,a'} \]  
(B2.24)
for the spring survey, and
\[ \hat{P}_{y,d}^{\text{surv}} = \sum_{a} P_{y,a}^{\text{surv}} \hat{A}_{a,d}^{\text{mid}} \]  
(B2.25)

for the fall survey,
where \( A_{a,d}^{\text{str}} \) and \( A_{a,d}^{\text{mid}} \) are the proportions of fish of age \( a \) that fall in the length group \( l \) (i.e.,
\[ \sum_{l} A_{a,d}^{\text{str}} = 1 \] and \( \sum_{l} A_{a,d}^{\text{mid}} = 1 \) for all ages) at the beginning of the year and at the middle of the year respectively.

The matrices \( A_{a,d}^{\text{str}} \) and \( A_{a,d}^{\text{mid}} \) are calculated under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:
\[ L_a^{\text{str}} \sim N\left[L_\infty \left(1 - e^{-\kappa (a-t_o)}\right)\right] \left(\theta_a^{\text{str}}\right)^2 \]  
(B2.26)
for the spring survey and
\[ L_a^{\text{mid}} \sim N\left[L_\infty \left(1 - e^{-\kappa (a+0.5-t_o)}\right)\right] \left(\theta_a^{\text{mid}}\right)^2 \]  
(B2.27)
for the fall survey,
where
\[ \theta_a^{\text{str}} \] and \( \theta_a^{\text{mid}} \) are the standard deviation of begin and mid-year length-at-age \( a \) respectively, which are modelled to be proportional to the expected length-at-age \( a \), i.e.:
\[ \theta_a^{\text{str}} = \beta \left[L_\infty \left(1 - e^{-\kappa (a-t_o)}\right)\right] \]  
(B2.28)
and
\[ \theta_a^{\text{mid}} = \beta \left[L_\infty \left(1 - e^{-\kappa (a+0.5-t_o)}\right)\right] \]  
(B2.29)
with \( \beta \) an estimable parameter.

\[ L_\infty = 189 \text{ cm} \],
\[ \kappa = 0.0815 \text{ yr}^{-1} \],
\[ t_o = 0.0627 \text{ yr} \],

The following term is then added to the negative log-likelihood:
\[ -\elln \ L_{CAL}^{\text{surv}} = W_{\text{len}} \sum_{\text{surv}} \sum_{y} \sum_{l} \left( \elln \left( \sigma_{\text{len}}^{\text{surv}} / \sqrt{p_{y,l}^{\text{surv}}} \right) + p_{y,l}^{\text{surv}} \left( \elln p_{y,l}^{\text{surv}} - \elln \hat{P}_{y,l}^{\text{surv}} \right)^2 / 2 \sigma_{\text{len}}^{\text{surv}}^2 \right) \]  
(B2.30)
The \( W_{\text{len}} \) weighting factor may be set to a value less than 1 to downweight the contribution of the catch-at-length data (which tend to be positively correlated between adjacent length groups because the length distributions for adjacent ages overlap) to the overall negative log-likelihood compared to that of the CPUE data.

RCp does not incorporate any catch-at-length data.
B2.2.6. Stock-recruitment function residuals
The stock-recruitment residuals are assumed to be log normally distributed. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

\[ -\ell n L^{pen} = \sum_{y=1981}^{y_n} \left[ \frac{\varepsilon_y^2}{2\sigma_R^2} \right] \]  
(B2.31)

where

\[ \varepsilon_y \]  from \( N(0, (\sigma_R)^2) \),

\[ \sigma_R \]  is the standard deviation of the log-residuals, which is input.

Equation B2.31 is used when the stock-recruitment curve is estimated internally. In some analyses reported in this paper where BRP estimates are based on stock-recruitment curves estimated “externally” using the assessment outputs, this “stock-recruitment” term is included for the last two years only, simply to stabilize these estimates which are not well determined by the other data. In these cases, the \( \varepsilon_y \) are calculated as the deviations from the mean log recruitment for the ten preceding years, i.e. recruitment estimates for 2010 and 2011 are shrunk towards the geometric mean recruitment over the preceding decade.

B2.2.7. Catches

\[ -\ell n L^{catch} = \sum_{y} \left[ \frac{\ell n C_y - \ell n \hat{C}_y}{2\sigma_C^2} \right] \]  
(B2.32)

where

\( C_y \)  is the observed catch in year \( y \),

\( \hat{C}_y \)  is the predicted catch in year \( y \) (equation B2.7), and

\( \sigma_C \)  is the CV input: 0.5 for pre-1964 catches, 0.3 for catches between 1964 and 1981 and 0.1 for catches from 1982 onwards.

B2.2.8 Incorporation of Bigelow vs Albatross survey calibration
The survey data provided are adjusted for the years 2009 to 2011 which were obtained from Bigelow surveys; these have been adjusted to “Albatross equivalents” through use of calibration factors estimated independently from paired tow experiments (Miller et al., 2010). However the survey data before and after the switch of vessels also provide information on the calibration factors because they sample the same cohorts. Incorporation of this information in assessments in this paper has been effected by treating the estimate with its variance as a form of “prior” which is effectively updated in the penalised likelihood estimation when fitting the model. The following contribution is therefore added as a penalty (or a prior in a Bayesian contact) to the negative log-likelihood in the assessment:

\[ -\ln L^{calib} = (\Delta \ln \hat{q} - \Delta \ln q)^2 / 2\sigma_{\Delta\ln q}^2 \]  
(B2.33)

where

\( \Delta \ln q = \ln(2.235) \)  is the logged ratio of the catchability of the Bigelow to the Albatross, with standard error

\( \sigma_{\Delta\ln q} = 0.173 / 2.235 \),

\( \Delta \ln \hat{q} \)  is the logged ratio of the catchabilities, estimated directly in the fitting procedure, where

\( q_{\text{Big}}^{Spr/Aut} = e^{\Delta \ln \hat{q}} q_{\text{Alb}}^{Spr/Aut} \).

In RCp, the calibration parameters are fixed to those estimated by Miller et al. (2010).

B2.3. Estimation of precision
Where quoted, CV’s or 95% probability interval estimates are based on the Hessian.
**B2.4. Model parameters**

**B2.4.1. Fishing selectivity-at-age:**
For the NEFSC offshore surveys, the fishing selectivities are estimated separately for ages 1 to age 7. The estimated proportional decrease from ages 6 to 7 is assumed to continue multiplicatively to age 9+; this decrease parameter is bounded by 0, i.e. no increase is permitted.

The commercial fishing selectivity, $S_y$, is estimated separately for ages $a_{\text{min}}$ (1) to 6, and is taken to be flat thereafter. It is taken to differ over two periods: a) pre-1997, and b) 1998-present. The selectivities are estimated directly for each period.
B2.4.2. Other parameters

Stock-recruit standard dev.
\[ \sigma_R = 0.5 \]

Model plus group
\[ m = 9 \]

Commercial CAA
\[ a_{\text{minus}}^* = 1 \]
\[ a_{\text{plus}} = 7 \]

Survey CAA
NEFSC spr \nNEFSC fall
\[ a_{\text{minus}}^* = 1 \]
\[ a_{\text{plus}} = 7 \]

Natural mortality
\[ M = 0.2 \text{ and age independent} \]

Proportion mature-at-age
\[ f_u \text{ input, see Table B65} \]

Weight-at-age
\[ w_{y,u}^{\text{str}} \text{ input, see Table B39b} \]
\[ w_{y,u}^{\text{mid}} \text{ input, see Table B39a} \]

Initial conditions for a 1963 starting year
\[ \theta \text{ estimated} \]
\[ \phi \text{ estimated} \]

* Strictly not a minus group anymore since the catches at age zero are ignored.

B2.5. Biological Reference Points (BRPs)

It is possible to estimate BRPs internally within the assessment by fitting the stock-recruitment relationship directly within the assessment itself. The \( F_{\text{MSY}} \) estimate is obtained by using a bisection routine to find where the derivative of the equilibrium catch vs \( F \) relationship has a zero derivative. This has to be based on point estimates, so that the estimate of other BRPs are conditional on this point estimate of \( F_{\text{MSY}} \), with no Hessian based CV available for this quantity.

For some results reported here, however, the stock-recruitment relationships are fitted to the estimates of recruitment and spawning biomass provided by the various assessments to provide a basis to estimate BRPs. The rationale for estimation external to the assessment itself is to avoid assumptions about the form of the relationship influencing the assessment results. These fits are achieved by minimizing the following negative log-likelihood, where the \( e^{-\frac{\sigma^2}{2}} \) term is added for consistency with equation A1.4, i.e. the stock-recruitment curves estimated are mean-unbiased rather than median unbiased:
\[-\ln L = \sum_{y=2}^{2009} \left[ \frac{\ln(N_{y,1}) - \ln\left(\hat{N}_{y,1}e^{\frac{\sigma^2}{2}}\right)}{2\left(\sigma_R^2 + (CV_y)^2\right)} \right] \]

\hspace{1cm} (B2.34)

where

- $N_{y,1}$ is the "observed" (assessment estimated) recruitment in year $y$,
- $\hat{N}_{y,1}$ is the stock-recruitment model predicted recruitment in year $y$,
- $\sigma_R$ is the standard deviation of the log-residuals which is input (and set here to 0.5), and
- $CV_y$ is the Hessian-based CV for the "observed" recruitment in year $y$.

Note that the differential precision of the assessment estimates of recruitment is taken into account, and that the summation ends at 2009 because little by way of direct observation is as yet available to inform estimates of recruitment for 2010 and 2011.
Figure Appendix B3.1a. Trace for SSB in 1963 (top) and 2011 (bottom) for the initial chain. The trace shows some indication of incomplete mixing at the beginning of the chain for the earlier SSB estimate.
Figure Appendix B3.1b. Plot of autocorrelation within the initial chain of SSB in 1963 (top) and 2011 (bottom). This diagnostic suggests a much higher thinning rate is needed for the early estimates of SSB, while an addition thinning rate of 5 would probably suffice for more recent years.
Figure Appendix B3.2a. Trace for Freport in 1963 (top) and 2011 (bottom) for the initial chain. The trace shows some indication of incomplete mixing at the beginning of the chain for the earlier Freport estimate. Freport is the full fishing mortality on age 6.
Figure Appendix B3.2b. Plot of autocorrelation within the initial chain of Freport in 1963 (top) and 2011 (bottom). This diagnostic suggests a much higher thinning rate is needed for the early estimates of Freport, while an addition thinning rate of 5 would probably suffice for more recent years.
Figure Appendix B3.3a. Trace for SSB in 1963 (top) and 2011 (bottom) for the longer chain (10,000 iterations). The trace suggests adequate mixing.
Figure Appendix B3.3b. Plot of autocorrelation within the longer chain (10,000 iterations) of SSB in 1963 (top) and 2011 (bottom). This diagnostic suggests a slightly higher thinning rate is needed for the estimates of SSB.
Figure Appendix B3.4a. Trace for Freport in 1963 (top) and 2011 (bottom) for the longer chain (10,000 iterations). The trace suggests adequate mixing.
Figure Appendix B3.4b. Plot of autocorrelation within the longer chain (10,000 iterations) of Freport in 1963 (top) and 2011 (bottom). This diagnostic suggests a slightly higher thinning rate is needed for the estimates of Freport.
Figure Appendix B3.5a. Trace for SSB in 1963 (top) and 2011 (bottom) for the longer chain after burn-in and additional thinning (1,000 remaining iterations). The trace suggests adequate mixing.
Figure Appendix B3.5b. Plot of autocorrelation within the longer chain after burn-in and thinning (1000 remaining iterations) of SSB in 1963 (top) and 2011 (bottom). This diagnostic suggests no additional thinning is needed.
Figure Appendix B3.6a. Trace for Freport in 1963 (top) and 2011 (bottom) for the longer chain after burn-in and additional thinning (1,000 remaining iterations). The trace suggests adequate mixing.
Figure Appendix B3.6b. Plot of autocorrelation within the longer chain after burn-in and thinning (1000 remaining iterations) of Freport in 1963 (top) and 2011 (bottom). This diagnostic suggests no additional thinning is needed.
Figure Appendix B3.7. Comparison of distributions of numbers at age for the initial chain (200,000 thinned to 1000 iterations) and a longer chain (5 million, with burn-in and thinning to 1000 final iterations)
Figure Appendix B3.7 (cont.)
Figure Appendix B3.7 (cont.)
Appendix B4
ASAP sensitivity runs
Appendix Figure B4.1. Estimates of spawning stock biomass, fishing mortality and recruitment from a sensitivity run in which the starting year was changed from 1963-1982.
Appendix Figure B4.2. Estimates of spawning stock biomass, fishing mortality and recruitment from a sensitivity run in which the strata set used to calculate indices of abundance was changed from 01200-01300,01360-01400 (Base-Run) to 01010-01300,01360-01400 (Alternate Survey).